Goldstein 9.4

$$\begin{cases} Q = \ln\left[\frac{1}{q}\sin p\right] \\ P = q\,\frac{\cos p}{\sin p} \end{cases}$$

$$e^Q = \frac{1}{q}\sin p \Rightarrow \sin p = q e^Q, \quad P = e^{-Q}\cos p$$

The third generating function relation gives

if $F_3 = F_3(p, Q, t) + q_i p_i$, then $q_i = -\frac{\partial F_3}{\partial p_i}$

$$P_i = -\frac{\partial F_3}{\partial Q_i}$$

We use this ansatz and set up a differential eq for F_3:

$$\begin{cases} q = -\frac{\partial F_3}{\partial p} = e^{-Q}\sin p \\ P = -\frac{\partial F_3}{\partial Q} = e^{-Q}\cos p \end{cases}$$

This is solved via $\boxed{F_3 = e^{-Q}\sin p}$, so the given transformation satisfies the canonical transformation eq.

Davidson Cheng

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